THIS IS A VERY COMMON EQUATION USED TO DESCRIBE DECOMPOSITION, DECAY, DEATH RATE,

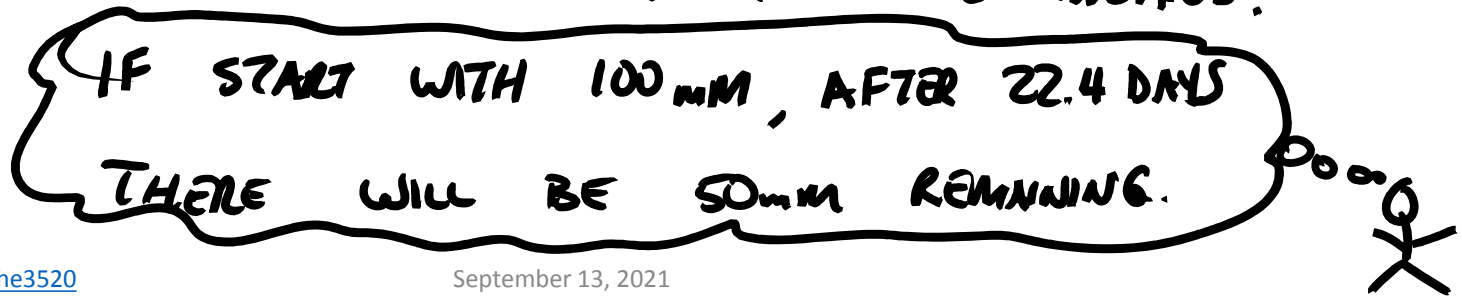
"EXPONENTIAL DECAY"

EXAMPLE 2

A CERTAIN RADIOISOTOPE HAS A HALF-LIFE OF 22.4 DAYS. WHAT WILL THE CONCENTRATION OF A 150 mmol/L SOLUTION BE AFTER 70 DAYS?

$$C_A = C_{A0} e^{-kt}$$

THINK ABOUT WHAT "22.4 DAYS HALF-LIFE" MEANS.



$$C_A = C_{A0} e^{-kt}$$

$$50 = 100 e^{-k(22.4 \text{ days})}$$

$$k = 0.0309 \text{ days}^{-1}$$

NOW, SOLVE ORIGINAL PROBLEM....

$$C_A = 150 e^{-(0.0309)(70)}$$

$$C_A = 17.2 \text{ mmol/L}$$

EXAMPLE 3

A CERTAIN POLLUTANT (M=192) DECOMPOSES BY FIRST ORDER KINETICS WITH A RATE CONSTANT OF $5.20 \times 10^{-3}\ day^{-1}$. IF 1000Kg ARE SPILLED IN A 6.5×10^{9} L POND, HOW LONG WILL IT TAKE FOR THE CONCENTRATION OF POLLUTANT IN THE POND TO FALL BELOW 5.0 ppb?

1 Kg ≈ 1 L water

1 mg/L = 1 ppm

1 μg/L = 1 ppb

$$C_A = C_{A0}\, e^{-k_1 t}$$

k_1: GIVEN

t: SOLVE FOR!

C_{A0}: CAN CALCULATE FROM INFO (mass & volume)

C_A: Relates to 5.0 ppb

$$C_{A0} = \frac{1000\,Kg}{6.5\times10^{9}\,L} \times \frac{1000\,g}{Kg} \times \frac{mol}{182\,g} = 8.453\times10^{-7}\ \frac{mol}{L}$$

$$C_{A0} \Rightarrow \quad 5_{ppb} = \frac{5.0\,\mu g}{L} \times \frac{g}{10^{6}\,\mu g} \times \frac{mol}{182\,g} = 0.274\times10^{-7}\ \frac{mol}{L}$$

$$C_A = C_{A0} e^{-kt}$$

$$0.274 = 8.453\, e^{(-5.20\times10^{-3})(t)}$$

$$\boxed{t = 659 \text{ days}}$$

$$\cong 1.81 \text{ years}$$

2) FIRST ORDER IRREVERSIBLE SINGE REACTION USING FRACTION CONVERSION

$$\chi_A = \frac{n_{A0} - n_A}{n_{A0}}$$

$$\chi_A = \frac{C_{A0} V_0 - C_A V}{C_{A0} V_0}$$

IF VOLUME IS CONSTANT, $V = V_0$

AND $$\boxed{\chi_A = \frac{C_{A0} - C_A}{C_{A0}}}$$ OR $$\boxed{C_A = C_{A0}(1 - \chi_A)}$$

THIS RELATIONSHIP CAN BE USED IN DIFFERENTIAL EQUATION.

$$\frac{dC_A}{dt} = -kC_A$$

SUBSTITUTION

$$\frac{d}{dt}\left[C_{A0}(1-X_A)\right] = -k\,C_{A0}(1-X_A)$$

$$\frac{d}{dt}(1-X_A) = -k(1-X_A)$$

$$\frac{d}{dt}(1) - \frac{d(X_A)}{dt} = -k(1-X_A)$$

$$\frac{dx_A}{dt} = k(1-x_A)$$

$$\int_0^{x_A} \frac{dx_A}{1-x_A} = k\int_0^t dt$$